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THE MONOPHONIC DIAMETRAL PATH FIXING MONOPHONIC NUMBER OF A GRAPH

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ABSTRACT

For a connected graph $G = (V, E)$, let P be amonophonic diametral path of G. A set $M \subseteq V(G) - V(P)$ is called aP −monophonic set of G if every vertex of G lies on a $x - y$ monophonic pathwhere $x \in P$and $y \in M$. The minimum cardinality of a $P-$ monophonic set of G is$P$ −monophonic number of Gdenoted by $m_p(G)$. A monophonic set of cardinality$m_p(G)$ is called a m_p −set of$G$. P −monophonic number of certain classes of graphs are studied. Connected graphs of order p with P −monophonic number 0 and $p - 2$ are characterized. It is shown that for integers with $2 \leq a \leq b$, there exists a connected graph G of order p, with $m(G) = a$and $m_P(G) = b$.

Keywords: *monophonic path, monophonic number,P-monophonic number*
AMS subject classification: 05C38

I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic graph theoretic terminology we refer to Harary [1,3].A *chord* of a path $u_0, u_1, u_2, \ldots\ldots, u_n$ is an edge $u_i u_j$ with $j \geq i+2, (0 \leq i, j \leq n)$. An $u - v$ path P is called a *monophonic path* if it is a chordless path . For two vertices u and v in a connected graph G, the *monophonic distance* $d_m(u, v)$ is the length of the longest $u - v$ monophonic path in G. A $u - v$ monophonic path of length $d_m(u, v)$ is called a $u - v$ *monophonic*. For a vertex v of G, the *monophonic eccentricity* $e_m(v)$ is the monophonic distance between v and a vertex farthest from v. The minimum monophonic eccentricity among the vertices is the *monophonic radius*, $rad_m(G)$ and the maximum monophonic eccentricity is the *monophonic diameter* $diam_m(G)$ of G.The monophonic distance of a graph is introduced in [4]. A *monophonic set* of G is a set $M \subseteq V$ such that every vertex of G is contained in a monophonic path joining some pair of vertices in M. The *monophonic number*$m(G)$ of G is the minimum order of its monophonic sets and any monophonic set of order $m(G)$ is a *minimum monophonic set* or simply am −set of G.The monophonic number of agraph is introduced in [2] and further studied in [5,6,7]. These concepts have many applications in location theory and convexity theory. There are interesting applications of these concepts to the problem of designing the route for a shuttle and communication network design. We further extend these concepts to the monophonic diametral path of G and present several interesting result.
The following theorem is used in sequel.

Theorem 1.1.[5] Each extreme vertex of a graph G belongs to every monophonic set of G.

II. THE DIAMETRAL PATH FIXING MONOPHONIC NUMBER OF A GRAPH.

Definition 2.1. Let G be a connected graph and P be amonophonic diametral path of G. A set $M \subseteq V(G)$ is said to be aP −*monophonic set* of G if every vertex of G lies on a monophonic path joining a vertex of M and a vertex of$P$. The P −*monophonic number*$m_p(G)$ of G is the minimum order of its P −monophonic sets and any $P -$ monophonic set of order $m_p(G)$ is a *minimum*P −*monophonic set* or simply m_p −set.

Example 2.2. For the graph G given in Figure 2.1, for the monophonic diametral path $P: v_1, v_2, v_8, v_6, v_4, v_5$, $M = \{v_3\}$ is a m_p −set of G so that $m_p(G) = 1$. Also for the monophonic diametral path $P: v_1, v_2, v_7, v_6, v_4, v_5$, $M_1 = \{v_3, v_8\}$ is a m_p −set of G so that $m_p(G) = 2$.

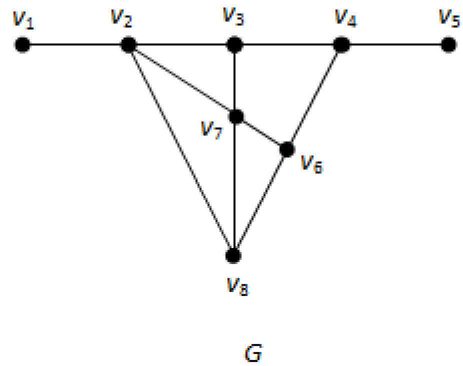

Figure 2.1

Theorem2.3. Let $x - y$ be a monophonicdiametral path P of a connected graph G of order at least 3. Then every extreme vertex (whether x or y is extreme or not) belongs to every P −monophonicset of G.

Proof. Let $x - y$ be a monophonic diametral path P and M be a P −monophonic set of G and v be an extreme vertex of G such that $v \neq x, v \neq y$. Let $\{v_1, v_2, \ldots, v_k\}$ be the neighboursof v. Suppose that $v \notin M$. Since M is a P −monophonic set of G, v lies on a P −monophonic path $Q: x = x_1, x_2, \ldots, v_i, v, v_j, \ldots, x_m = y$. where $x, y \in M$. Since $v_i v_j$ is chord of Q and so $Q$is not a monophonic path, which is a contradiction.Therefore $v \in M$. ■

Theorem 2.4. Let G be a connected graph with cut-vertices, P be a monophonic diametral path of G, and M be a P-monophonic set of G. If v is a cut-vertex of G such that $v \notin V(P)$, then every component of $G - v$ contains an element of M.

Proof . Suppose that there is a component $G1$ of $G - v$ such that $G1$ contains no vertex of M. By Theorem 2.3, $G1$ does not contain any end-vertex of G. Thus G_1 contains at least one vertex, say u. Since M is a P − monophonic set, there exists vertices $x \in M$ and $y \in P$ such that u lies on the $x - y$monophonic path $Q : x = u_0, u_1, u_2, \ldots, u, .., u_t = y$in G. Let Q_1 be a $x - u$ sub path of Q and Q_2be a $u - y$ subpath of Q. Since v is a cut-vertex of G, both Q_1 and Q_2 contain v so that Q is not a path, which is a contradiction. Thus every component of $G - v$ contains an element of$M$. ■

Theorem 2.5. Let G be a connected graph and P be a monophonic diametral path of G. Then no cut-vertex of G belongs to any minimum P −monophonic set of G.

Proof. Let M be a minimum P − monophonic set of G and $v \in M$ be any vertex. Then $v \notin V(P)$. We claim that v is not a cut vertex of G. Suppose that v is a cut vertex of G. Let $G_1, G_2, \ldots, G_r (r \geq 2)$ be the components of $G - v$. By Theorem 2.3, each component $G_i (1 \leq i \leq r)$ contains an element of M. We claim that $M_1 = M - \{v\}$ is also P − monophonic set of G. Let x be a vertex of $V(G) - V(P)$. Since M is a P −monophonic set of G,x lies on a monophonic path Rjoining a pair of vertices u of $M$and v of P. Assume without loss of generality that $u \in G_1$. Since v is adjacent to at least one vertex of each $G_i (1 \leq i \leq r)$, assume that v is adjacent to z in $G_k, k \neq 1$. Since M is a P −monophonic set, z lies on a monophonic path Q joining v and a vertex w of M such that w must necessarily belongs to G_k. Thus $w \neq v$. Now, since v is a cut vertex of $G, R \cup Q$ is a path joining u and w in $M$and thus the vertex x lies on this monophonic path joining two vertices u and $w$of M_1. Thus we have proved that every vertex that lies on a monophonic path joining a pair of vertices u and v of M also lies on a monophonic path joining two vertices of M_1. Hence it follows that every vertex of G lies on a monophonic path joining two vertices of M_1, which shows that M_1 is a P −monophonic set of G. Since $|M_1| = |M| - 1$, this contradicts the fact that M is a minimum monophonic set of G. Hence $v \notin M$ so that no cut vertex of G belongs to any minimum P −monophonic set of G. ■

Corollary 2.6. For any non-trivial tree T, $m_P(T) = k - 2$, where$k$ is number of end vertices of T.
Proof. This follows from Theorems 2.3 and 2.5. ■

Corollary 2.7. For the complete graph $G = K_p(p \geq 2), m_P(Kp) = p - 2$.
Proof. Let $P: x, y$ be a monophonic diametral pathof K_p.Since every vertex of the complete graph, $K_P(p \geq 2)$ is an extreme vertex, the set $V(G) - \{x, y\}$ is the unique P −monophonic set of K_p. Thus $m_P(K_p) = p - 2$. ■

Theorem 2.8. For a cycle $C_p, p \geq 4, m_P(C_p) = 1$.
Proof. Let P be a monophonic diametral path of $G$and $x \in V(G) - V(P)$. Then $M = \{x\}$ is a P −monophonic set of G so that $m_p(C_p) = 1$. ■

Theorem 2.9. For $G = K_{m,n}, m_p(G) = \begin{cases} 2, & 4 \leq m \leq n \\ 1, & m = 2 \text{ or } 3, n \geq 2 \end{cases}$
Proof. Let $u = u_1, u_2, \ldots, u_m$ and $v = v_1, v_2, \ldots, v_n$ be a bi-partite set of G. If $m = 2$ or $3, n \geq 2$, it is easily verified that$m_p(G) = 1$. So let $4 \leq m \leq n$.Let $P = u_1, v_1, u_2$be a monophonic diametral path of $G$and $M = \{u_3, v_2\}$. Then it is clear that M is a P −monophonic set of Gso that $m_p(G) \leq 2$ we prove that $m_p(G) = 2$. It is clear that $m_p(G) \geq 1$. Suppose that $m_p(G) = 1$. Then there exists a P −monophonic set M'such that $|M'| = 1$. Let $M' = \{u_i'\}$. Then there exists $u_j \in U$ such that $u_j' \notin U$. It is clear that this u_j'not belongs to any monophonic joining vertex of P and a vertex of M'. If $M' = \{v_i'\}$ then there exists the $v_j' \in M'$. It is clear that v_j' not belongs to any monophonic joining vertex of P and a vertex ofM', which is a contradiction. Therefore $m_p(G) = 2$.

Theorem 2.10. For any connected graph G of order $p, 0 \leq m_P(G) \leq p - d_m - 1$.
Proof. Any P −monophonic set is either empty or contains at least one vertex, it is clear that $m_p(G) \geq 0$. If $G = K_p$, then $m_p(G) = p - 2$ so that the result is trivial. Assume that G is non complete. Let u and v be two vertices of G such that $d_m(u, v) = d_m \geq 2$. Let $P: u = v_0, v_1, v_2, \ldots, v_{d-1}, v_d = v$ be a monophonicdiamateral path. Let $M = V(G) - V(P)$. Then M is P −monophonicset of G. Hence $m_P(G) \leq p - d_m - 1$. ■

Remark 2.11. The bounds in Theorem 2.10 are sharp. For the path $G = P_p, m_p(G) = 0$ and for the complete graph $= K_p, m_p(G) = p - 2$. Also the bounds in Theorem 2.10 can be strict. For the graph G given in Figure 2.1, for the monophonic diametral path $P: v_1, v_2, v_7, v_6, v_4, v_5$, $d_m = 5, p = 8$ and $m_p(G) = 2$ so that $0 < m_P(G) < p - d_m - 1$.

Theorem 2.12. Let G be a connected graph. Then $m_p(G) = p - 2$ if and only if $G = K_p$.
Proof. First assume that $G = K_p$. Then by Corollary 2.7,$m_p(G) = p - 2$. Suppose that G is non-complete then $d_m \geq 2$. By Theorem 2.10,$m_p(G) \leq p - 3$, which is a contradiction. Therefore $G = K_p$. ■

Theorem 2.13. For any connected graph $G, m(G) \leq m_p(G) + d_m + 1$.
Proof. Let P be a $x - y$ monophonic diametral path of G and M be a P −monophonic set of G. Then $M \cup V(G)$ is a monophonic set of G, it follows that $m(G) \leq m_p(G) + d_m + 1$. ■

Theorem 2.14. For every integers with $2 \leq a < b$, there exists a connected graph Gmonophonic diameter d_m such that$m(G) = a$and $m_p(G) = b$.
Proof. Let $P_{d_m}: u_0, u_1, \ldots, u_{d_{m-1}}, u_{d_m}$ be a path of length d_m.Now, add $(a - 1)$ new vertices $v_1, v_2, \ldots, v_{a-1}$ to P_d and join each to $u_1$and u_0, thereby producing a graph$H$. Then add
$b - a + 1$ new vertices $w_1, w_2, \ldots, w_{b-a+1}$to H and join each to both u_0 and u_2, Also join each $w_i$and w_j $i \neq j$

obtaining the graph G of Figure 2.2 so that G has monophonic diameter d_m. Let $M = \{u_{d_m}, v_1, v_2, \ldots, v_{a-1}\}$ be the set of extreme vertices of G. By Theorem 1.1, M is contained in is contained in every monophonic set of G and so $m(G) \geq a$. It is clear that M is a monophonic set of G so that $m(G) = a$. Next we show that $m_p(G) = b$. Now$P: u_0, u_1, \ldots, u_{d_{m-1}}, u_{d_m}$ is a monophonic diameteral path of length d_m. Let$M_1 = M - \{u_{d_m}\}$.Then by Theorem 2.3, M_1 is a subset of every P −monophonic set of G.It is clear that M_1 is not a P −monophonic set of G.It is easily observed that $w_i(1 \leq i \leq b - a + 1)$ belongs to every P −monophonic set of G and so$m_p(G) \geq b$. Now, $M_2 = M_1 \cup \{w_1, w_2, w_{b-a+1}\}$ is a P −monophonic set of G so that$m_p(G) = b$. ■

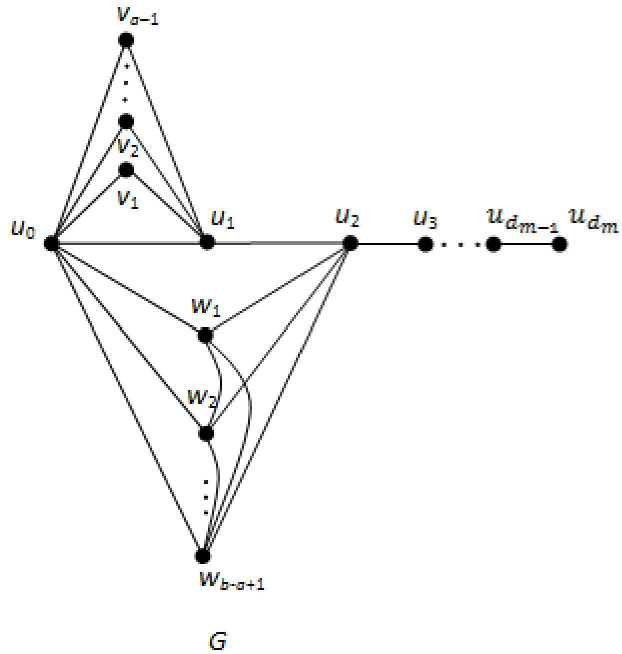

Figure 2.2

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